

Algebra problem-solving and end-of-year review

CURRICULUM ALIGNMENT

ALG.PRR.4a

identify, explain and apply generalisations, including properties of operations, mathematical models and patterns.

ALG.EXP.4

articulate, represent and solve mathematical situations through the use of expressions and equations that include letter-symbols.

ALG.PRR.4b

represent mathematical structures in multiple ways, including verbal expressions, diagrams and symbolic representations.

INTERACTIVES **Function Machine** · display, explore

WHAT THIS LESSON TEACHES

Finding the rule that links a term's position to its value lets you jump to any term without listing them all.

→ 1 to 4, 2 to 7, 3 to 10 gives 'multiply the position by 3 then add 1'.

→ $3n + 1 = 34$ becomes $3n = 33$ then $n = 11$.

MODEL THIS ON THE BOARD

SOLVE THE PATTERN 5, 9, 13, 17 TO REACH 61

- 1 Write the rule in words: multiply position by 4 then add 1.
- 2 Write the expression: $4n + 1$.
- 3 Set up the equation: $4n + 1 = 61$.
- 4 Take away 1: $4n = 60$.
- 5 Share by 4: $n = 15$.

DISCUSSION STARTER

REFLECT Where this year did using a letter for an unknown make a problem easier?

LESSON ARC

Open with the growing pattern 4, 7, 10, 13 and the 100th-term question — take hands-up guesses before revealing anything. Two function machines on the board make the pivot: the rule that grows a pattern becomes the equation that solves for a term. Together the class works one full challenge (5, 9, 13, 17 → find which position gives 61), then pupils label all five stages in their copy. Class Challenge is a five-stage mystery worked in the copy with answers confirmed aloud.

TEACHING MOVES

1. **Getting Started.** Put up 4, 7, 10, 13 and ask for the 100th term with hands up — take two or three answers. When someone says 'count on in threes', push back: 'a hundred times? Is there a faster way?' Hold out for the idea of a rule; don't confirm it yet.
2. **Watch and Notice.** On the first machine, before the arrow lands, ask 'why three and not another number?' — draw out that the step is three. Test it: $3 \times 4 + 1 = 13$, matches. On the second machine make the pivot explicit: 'forwards we put a position in; backwards we know the value and hunt for the position.' Walk the reverse working slowly, pointing at each step, and ask 'why take away the 1 before we share by 3?'

3. **Try It Together.** Pupils take turns at the board; the class agrees or corrects aloud. Feed positions 1, 2, 3 through the machine so the step of 4 and start of 5 are visible, then build $4n + 1$ with the pen moving. Flip it to reach 61 and ask 'which did the machine do last?' to head off pupils undoing the $\times 4$ first.
4. **Work the Stages in Your Copy.** Walk the room checking each stage is labelled — table, rule, expression, equation, solution — and that the undoing runs in the right order. This is whole-class copybook practice, not marking. If you see $4n = 61 - 1$ muddled, prompt 'which step did the machine do last, and so which do we undo first?'
5. **Class Challenge.** Brisk practice: pupils work the five stages in their copy and the class confirms each answer aloud before the next. The 'unlocks' wording is just narrative — there's no on-screen gate. Fast finishers draft their own five-stage mystery quietly while the rest catch up.
6. **What Did We Notice?.** Listen for the shift from 'continuing the list' to 'having a rule' and revoice a strong answer: 'so the letter lets us leap straight to any term, forwards or backwards.' Land it on a real-life rule — bus fare as base plus so-much-per-stop, or points-per-goal in a match.

COMMON MISCONCEPTIONS

⚠ For $4n + 1 = 61$, pupils undo the multiply first: $61 \div 4$, then subtract — reversing the two steps in the wrong order.

Point back at the function machine and ask 'which did the machine do last — times four, or add one?' The machine added last, so we undo the adding first. Feed the answer back through the machine to check it lands on 61.

⚠ Reading the pattern's first term as the multiplier: pupils say the rule for 5, 9, 13, 17 is ' $\times 5$ ' because it starts at 5, ignoring that it steps up by 4.

Feed positions 1, 2, 3 through the explore machine so the class watches the value jump by 4 each time. The step is the multiplier; the first term tells you the add-on. Separate the two questions: 'what's the step?' then 'what's added on to reach the first term?'

DIFFERENTIATION

EMERGING

- Stay on the machine's forward direction — pupils substitute positions to find terms and skip the reverse equation until they're secure going forwards.
- Give the copybook stages with the column headings (Table / Rule / Expression / Equation / Solution) already written so pupils fill values, not invent the layout.

DEVELOPING

- After the copybook challenge, change the target from 61 to a value that isn't in the pattern (e.g. 60) and ask what n comes out as — and whether that means 60 is really a term.
- Ask pupils to write the rule for a pattern that goes down, e.g. 20, 17, 14, 11, and check the machine still works.

PROFICIENT

- In the Class Challenge, have fast finishers write their own five-stage mystery from a pattern of their choice, then narrate one stage aloud for the class to solve.
- Pose: can you find a pattern whose rule is $5n - 2$? Work backwards from the expression to the first four terms and justify each.

✂ **Cross-curricular:** Tie to real spending — a taxi charge of a €4 flag-fall plus €2 a kilometre is exactly $2n + 4$; pupils find the cost of a 7 km trip.

ANSWER KEY

Scaffold: visual check on the page

Q1: 74

Q2: 34

Q3: 301

Q4: 21

a) Always : Each term adds 3, so $3n + 1$ fits every position.

b) Always : The rule $3n + 1$ gives the 100th term straight away: 301.

c) Never : That pattern uses $4n + 1$ instead.

d) Sometimes : True for $3n + 1$; false for $2n$, which takes one step.

EXTENSION SHEET · STRETCH ANSWERS

S1: -6

S2: + 13

S3: + 14

S4: 43

S5: 53