

Angle sum of a quadrilateral

CURRICULUM ALIGNMENT

SHA.SHP.4b

investigate and construct angles in the context of shape; and solve angle-related problems.

ALG.PRR.4a

identify, explain and apply generalisations, including properties of operations, mathematical models and patterns.

INTERACTIVES Polygon Angles · challenge, display, explore

WHAT THIS LESSON TEACHES

The four interior angles of any quadrilateral always add to 360 degrees.

$$\rightarrow 90 + 90 + 90 + 90 = 360$$

$$\rightarrow 100 + 80 + 100 + 80 = 360$$

MODEL THIS ON THE BOARD

FINDING THE MISSING ANGLE

- 1 Split the quadrilateral into two triangles by drawing one diagonal.
- 2 Each triangle sums to 180 degrees so together they make $2 \times 180 = 360$ degrees.
- 3 Add the three known angles then subtract from 360 to find the fourth.

LESSON ARC

Open problem-first: sketch a quadrilateral with one diagonal ruled across it and ask pupils to predict the angle sum from what they know about triangles — write predictions up unconfirmed. Move through three snapshot shapes, saving the irregular one to clinch that the total stays 360°. Pupils rule their own diagonal in the copybook and label 180° in each triangle. The drag-corner interactive shows angles trading off while the sum holds, then the Class Challenge works four missing-angle problems on the board.

TEACHING MOVES

1. **Getting Started.** Give five seconds of silent think-time before hands go up, then take three predictions for the four-angle total and write all three on the board without confirming any. Listen for 'two triangles, so two lots of 180' — hold that pupil's reasoning to pull back out at the reveal, don't validate it yet.
2. **Watch and Notice.** Point at each of the three snapshots in turn rather than pre-explaining. Let the class total the two right-angled shapes in their heads; then land on the irregular third one — nobody can guess it, yet it still reads 360°. Only now name the rule, quoting back the hook pupil: one diagonal, two triangles, 180 each, so 360.
3. **Split and Show It in Your Copy.** Walk the room checking the diagonal genuinely reaches an opposite corner — some pupils clip off a corner instead. This beat is construction only: they rule accurately with a ruler, then label 180° in each triangle and write $2 \times 180^\circ = 360^\circ$. No missing-angle sums here.
4. **Try It Together.** Bring individual pupils up to drag a corner while the rest predict which angle will shrink before the drag. The move that lands it: stretch one angle huge and watch another shrink as the sum readout holds at 360°. Revoice: 'so the angles trade off, but the pot is always 360.'
5. **Class Challenge.** Brisk practice — roughly two minutes per problem, four board turns inside the slot, class confirms each answer before moving on. Cue the method every time: 'add the three you know, then

take that from 360.' On the fourth problem watch for pupils who add $85 + 95 + 130$ wrongly rather than misusing the method.

6. **What Did We Notice?** Ask why two triangles guarantees 360° , and revoice a strong answer into one clean sentence so the whole class hears the reasoning chain. For the pentagon predict-forward, take a couple of guesses and leave it open — don't confirm the 540° .

COMMON MISCONCEPTIONS

⚠ A pupil rules a line that clips off one corner rather than joining two opposite corners, so they get a triangle plus a smaller quadrilateral instead of two triangles.

Catch it at the copybook step. Put your finger on the two corners that must be joined and have the pupil re-rule from corner to opposite corner. Ask 'how many triangles now?' — the two-triangle split is what makes the 2×180 argument work.

⚠ In the Class Challenge a pupil gets the fourth angle wrong not because of the method but because they add the three known angles incorrectly — e.g. $85 + 95 + 130$ comes out short.

Separate the two steps out loud: 'the method is right — let's just re-add the three we know.' Redo the addition column-wise on the board, then subtract from 360. This keeps their confidence in the 360° rule intact while fixing the arithmetic slip.

⚠ A pupil thinks the 360° total only works for the neat right-angled shapes and that the irregular quadrilateral must be different.

Go back to the drag-corner interactive and stretch one angle right out while the sum readout stays on 360° . The shape bends wildly but the pot never changes — 'the angles trade off, the total holds.'

DIFFERENTIATION

EMERGING

- Let these pupils copy a ready-drawn quadrilateral with its diagonal already dotted in, so they only rule over the line and label 180° in each triangle rather than inventing the shape.
- In the Class Challenge, hand them the first problem (three 90s) — the square total is one they can reason without heavy arithmetic.

DEVELOPING

- After the copybook step, ask them to draw a second quadrilateral and rule the diagonal from the other pair of corners — does the two-triangle split still give 360° ?
- Give a missing-angle problem where two angles are given and two are equal (e.g. 110° , 130° , and two matching angles) — they must halve the remainder.

PROFICIENT

- Pose the pentagon question early: split a five-sided shape from one corner, count the triangles, and predict the angle sum — then justify why it's $3 \times 180^\circ$. Pair this with a copybook sketch.
- Ask them to write, in their own words for a younger pupil, why any four-sided shape totals 360° no matter how squashed it looks.

✂ **Cross-curricular:** Tie to visual arts — pupils sketch a tiling pattern of quadrilaterals and check the four angles meeting at a point relate to the 360° around a full turn.

ANSWER KEY

a) 360°

b) 85°

c) 110°

Q1: 30°

Q2: 100°

Q3: 50°

Q4: 125°

Q5: 75°

a) **Always** : This is the rule for any four-sided shape.

b) **Never** : They always add to exactly 360 degrees.

c) **Always** : Two triangles each sum to 180 degrees.

d) **Sometimes** : True for a square or a rectangle; false for a parallelogram or a kite.

EXTENSION SHEET · STRETCH ANSWERS

S1: 11°

S2: 99°

S3: 37°

S4: 30°

S5: 106°

Investigation: Angle sums of polygons — open-ended; scan pupils' working for valid solution paths rather than a single answer.