

Lesson 111 ALGEBRA

Properties of operations: commutative, associative, distributive

CURRICULUM ALIGNMENT

ALG.PRR.4a

identify, explain and apply generalisations, including properties of operations, mathematical models and patterns.

NUM.OPS.4

build upon, select and make use of a range of operation strategies.

INTERACTIVES

Area Model · display

Balance Scales · challenge, display, explore

WHAT THIS LESSON TEACHES

For addition and multiplication the order or grouping of the numbers does not change the answer. You can also break a multiplication into easier parts with the distributive property.

→ 4×7 and 7×4 both equal 28.

→ $(2 + 3) + 5$ and $2 + (3 + 5)$ both equal 10.

MODEL THIS ON THE BOARD

DISTRIBUTIVE TRICK ON THE BOARD

- 1 Start with the fact 4×6 .
- 2 Rewrite 6 as $5 + 1$ so $4 \times (5 + 1)$.
- 3 Split into two easy multiplications: 4×5 and 4×1 .
- 4 Add the parts: $20 + 4 = 24$.

LESSON ARC

Open with 4×7 and 7×4 left on the board; take three hands-up answers on whether they match, holding for why without confirming. Walk the balance-scales interactive through commutative, associative and distributive pans, then tip $8 - 3$ against $3 - 8$ so the class sees order matters for subtraction. Pupils test each property together on the beam, write one own example under each copybook heading, then build the empty pan in Class Challenge so the beam stays level — distributive rounds turn a hard fact into two easier ones.

TEACHING MOVES

1. **Getting Started.** Leave both multiplications on the board and take three hands-up answers only — no open call-outs. Most will say equal; hold out for why without confirming. The balance beam in the next step is how the class settles it.
2. **Watch and Notice.** Start with 4×7 against 7×4 ; pause before the second pan and take a level-or-tip prediction. Name commutative aloud when both pans hold 28. On the associative beam, nominate a pupil to name the property before you reveal. Give the distributive beam the most time — say ‘four sixes is the same as four fives and four ones’ and ask why each part inside the bracket must be multiplied before adding. Finish with $8 - 3$ against $3 - 8$ so the tip shows swapping subtraction does not keep the sides equal.
3. **Try It Together.** Run explore mode on the single beam, resetting each round: 5×3 against 3×5 , then the associative regroup, then $4 \times (5 + 1) = 4 \times 5 + 4 \times 1$. Take a predicted level-or-tipped before each load. Linger on the distributive round and watch for pupils who think the brackets change the answer.

4. **Test the Properties in Your Copy.** Walk the room glancing at the three headings — this is whole-class copybook practice, not marking. Check both sides of each pupil's own example match. Prompt any pupil whose distributive example only multiplies one part of the bracket.
5. **Class Challenge.** Keep board turns brisk: pupil builds the empty pan, class confirms, press Check, move on. On the round showing $6 \times 5 + 6 \times 2$, ask aloud what the two parts add up to and what single fact that gives — so they hear six fives and six twos make seven sixes.
6. **What Did We Notice?.** Listen for a hard fact such as 7×8 split into a friendly pair like $7 \times 5 + 7 \times 3$. Re-voice: 'so you turned one hard fact into two you already know.' Head off the idea that brackets change the total — they only change how we get there.

COMMON MISCONCEPTIONS

⚠ Pupils write $4 \times (5 + 1)$ as $4 \times 5 + 1$ — they multiply only the first part of the bracket and leave the second alone.

Freeze the beam on $4 \times (5 + 1) = 4 \times 5 + 4 \times 1$. Point at each part inside the bracket and ask 'what must the 4 hit?' Rebuild so both pans hold 24 before moving on.

⚠ Pupils say the brackets change the answer — ' $(2 + 3) + 5$ is different from $2 + (3 + 5)$ because the brackets moved.'

Reset the associative beam and load both pans. When both hold 10, re-voice: 'the brackets only change the path, not the total.'

⚠ Pupils swap a subtraction and expect the beam to stay level: ' $8 - 3$ should equal $3 - 8$ because order doesn't matter.'

Load $8 - 3$ against $3 - 8$ and let the beam tip. Ask what happens when you try to take 8 from 3 — name aloud that $-$ and $\div$ are the exceptions.

DIFFERENTIATION

EMERGING

- Stay on the commutative beam with single-digit products only until both pans match; hold the associative and distributive rounds as teacher-led watch-alongs.
- On the copybook page, supply the left side of each equation so the pupil only writes the swapped or regrouped right side and checks the total.

DEVELOPING

- After the copybook examples, ask: does $9 - 4$ equal $4 - 9$? Why does the commutative rule stop at $+$ and $\times$?
- On a distributive Class Challenge round, hide one factor and ask what must go in the empty pan to keep the beam level.

PROFICIENT

- Challenge them to split a hard tables fact (7×8 or 6×9) into two easier products on the beam and say the single fact it rebuilds.
- Ask whether $4 \times (6 - 1)$ still works the same way as $4 \times (5 + 1)$ — let them test on the beam and justify.

✂ **Cross-curricular:** At the school fair, total a pack of €6 items as $6 \times 5 + 6 \times 2$ rather than one hard 6×7 .

ANSWER KEY

a) 15 (5×3 and 3×5 both make 15)

b) 10

c) 24 (4×6 and $4 \times 5 + 4 \times 1$ both make 24)

Q1: yes, 72

Q2: Yes, 28

Q3: $80 + 16 = 96$

a) **Always** : The commutative property holds for every pair of numbers in multiplication.

b) **Never** : Subtraction is not commutative, as $8 - 3$ is not equal to $3 - 8$.

c) **Always** : The associative property always works for addition.

d) **Sometimes** : It helps when one factor is awkward, but is not needed for every easy fact.

EXTENSION SHEET · STRETCH ANSWERS

S1: distributive property

S2: distributive property

S3: distributive property

S4: associative property

S5: associative property