

Investigation: the staircase pattern

CURRICULUM ALIGNMENT

ALG.PRR.4a

identify, explain and apply generalisations, including properties of operations, mathematical models and patterns.

ALG.PRR.4b

represent mathematical structures in multiple ways, including verbal expressions, diagrams and symbolic representations.

INTERACTIVES Target Number · challenge

WHAT THIS LESSON TEACHES

In the staircase pattern each bigger size adds the next counting number of cubes. The totals are the running sums 1, then 1+2, then 1+2+3, and so on.

→ Size 3 needs 6 cubes

→ Size 4 needs 10 cubes because $6 + 4 = 10$

MODEL THIS ON THE BOARD

CUBES FOR A SIZE-6 STAIRCASE

- 1 Begin from the size-5 total of 15 cubes.
- 2 Add the next counting number 6: $15 + 6 = 21$.
- 3 A size-6 staircase is therefore built from 21 cubes.

LESSON ARC

Show the four staircase counts — 1, 3, 6, 10 — and take three genuine estimates for size 5 without confirming any. Build a running table on the board, drawing the jumps (+2, +3, +4) so pupils name the growing step themselves before you name the rule. Pupils sketch the four staircases in their copy and write the rule in words. Try It Together extends the table to sizes 7, 8 and 10; the Class Challenge is prediction practice plus a target round with the staircase numbers.

TEACHING MOVES

1. **Getting Started.** Take three named estimates for the size-5 count, not open call-outs, and write all three on the board unmarked. Say nothing about whether any is right — 'we'll come back and see whose guess held up.' Resist giving the rule here.
2. **Watch and Notice.** Write 1, 3, 6, 10 in a row and draw the jump arcs between them, saying each aloud — '+2, then +3, then +4.' Before drawing the next arc, ask 'the last jump was +4, so what comes next?' Only name the rule at the end, in pupils' own words: each staircase adds the next counting number.
3. **Sketch the Staircases in Your Copy.** Walk the room glancing at the count written under each sketch — this is practice, not marking. Watch for the rule pupils underline: push 'add the next counting number' over the vaguer 'it gets bigger', which won't let them predict size 10.
4. **Try It Together.** Pupils take turns at the board adding the next table row; the class agrees or corrects aloud. Before each total, ask them to predict from the rule (size 4 was 10, so +5 → 15; +6 → 21; +7 → 28). For the desk check use multilink to add columns onto the existing size-4 build rather than a fresh build, to protect timing.

5. **Class Challenge.** Brisk board practice — pupils predict sizes 4, 5, 8, 10, 12 (10, 15, 36, 55, 78), writing the running steps +5, +6, +7, +8 so the big sizes don't need a mental leap, then check each on the board before moving on. The last target in the round is the stretch; expect more than one route.
6. **What Did We Notice?.** Ask why the staircase keeps growing faster the taller it gets. Listen for pupils linking the widening gaps to the growing step, then revoice a strong answer: 'every new step is one cube taller than the one before, so the jumps keep getting bigger.'
7. **Write It in Your Journal.** Give quiet think-time for the paper journal entry. A pupil naming the multiplication tables as a contrast — constant step, not a growing one — is reasoning well; let that stand as a valid answer.

COMMON MISCONCEPTIONS

⚠ Pupils treat the step as a constant — they see the first jump is +2 and predict 3, 5, 7, 9 (adding 2 each time), so size 5 comes out as 9.

Go back to the board table and draw all three jumps at once — +2, +3, +4 — so the class sees the step itself changing. Ask 'is the jump the same each time, or does it grow?' Rebuild size 4 from size 3 with multilink so they see a whole extra column, not two cubes, gets added.

⚠ When jumping ahead to size 10 or 12, pupils lose track of which counting number to add and guess a total.

Stop and write out the running steps in a column: +5, +6, +7, +8... beside each new total. 'You can't skip — each staircase needs the exact next number.' Have the pupil at the board read the last step aloud before adding the next.

DIFFERENTIATION

EMERGING

- Stay with sizes 1 to 5 only in the copybook sketch while the class extends further — the pupil builds each staircase with multilink and counts the cubes rather than predicting.
- Give a pre-drawn running table with the jump arcs already marked so the pupil fills in the +2, +3, +4 without inventing the layout.

DEVELOPING

- After the Class Challenge prediction, ask: without the picture, work out size 15 — write every running step and check your total lands sensibly.
- Pose a missing-term question: 'a staircase has 28 cubes — what size is it?' and ask how they worked backwards.

PROFICIENT

- During the target round, narrate a harder variant: 'find two staircase numbers that add to 27, then a third route to the same total' — expect more than one answer and ask them to justify each.
- Challenge them to explain to a size-5-only pupil why the gaps widen, without using the word 'bigger' — force precise language about the growing step.

✂ **Cross-curricular:** Tie to Geography — the staircase counts are the same triangular numbers as rows of seats in an amphitheatre or GAA terrace; ask how many seats in 10 rows.

ANSWER KEY

a) 15, 21, 28

b) +5, +6, +7

c) 55, 78

Q1: 15

Q2: 15

Q3: 105

Q4: 36

Q5: 55

Q6: 78

a) **Always** : The sum $1+2+3$ is always 6.

b) **Always** : The jumps are always +2, then +3, then +4 and so on.

c) **Never** : Size 4 is always 10 cubes, never 9.

d) **Never** : Larger sizes always need more cubes in this pattern.

EXTENSION SHEET · STRETCH ANSWERS

S1: 22

S2: 38

S3: + 11

S4: 94

S5: 15