

Finding the rule from a table

CURRICULUM ALIGNMENT

ALG.PRR.4a

identify, explain and apply generalisations, including properties of operations, mathematical models and patterns.

ALG.PRR.4b

represent mathematical structures in multiple ways, including verbal expressions, diagrams and symbolic representations.

INTERACTIVES Function Machine · challenge, display, explore

WHAT THIS LESSON TEACHES

A hidden rule turns each input number into an output. Look at the pairs to work out what the machine does each time.

→ 2 goes in and 7 comes out; 5 goes in and 16 comes out so the rule is $\times 3$ then $+1$

→ 3 goes in and 10 comes out means the machine has turned 3 into 10

MODEL THIS ON THE BOARD

CRACKING THE PLAY MACHINE

- 1 Look at the pairs from the game machine: 2 turns into 7, 5 turns into 16, 10 turns into 31.
- 2 Test ideas on the first pair. $2 + 5 = 7$ works, but so does $2 \times 3 + 1 = 7$.
- 3 Try the next pair. $5 + 5 = 10$, yet the machine gave 16, so $+5$ is wrong.
- 4 Check $\times 3$ then $+1$: $5 \times 3 = 15$, $15 + 1 = 16$. It matches.
- 5 Check the last pair: $10 \times 3 = 30$, $30 + 1 = 31$. The hidden rule is $\times 3$ then $+1$.

LESSON ARC

Put the hidden-rule machine on the IWB with $2 \rightarrow 7$, $5 \rightarrow 16$, $10 \rightarrow 31$ and take a split of guesses without confirming any. In Watch and Notice, walk pupils through $+5$ failing on the second pair and the gap growing. Feed fresh inputs — especially 0 — through the Try It Together machine before naming the rule. Pupils crack Machine A in their copy, then the class works four machines in Class Challenge, ending on the trap pair that only $+5$ and $\times 4 + 2$ both fit.

TEACHING MOVES

1. **Getting Started.** Ten seconds of silent think-time, then take three hands-up answers and write all of them on the board unchanged. Say nothing about which is right — the split between $+5$ and $\times 3 + 1$ is the whole point, so leave the tension unresolved.
2. **Watch and Notice.** Point at each pair and narrate; don't read the rule off first. On the $\times 3 + 1$ machine, show $+5$ working for $2 \rightarrow 7$ then failing on $5 \rightarrow 16$, and say 'watch the gap grow' as the outputs pull away. On the $+7$ machine, let a pupil notice the $0 \rightarrow 7$ pair hands the add-number over instantly.
3. **Try It Together.** Feed 1, then 0, then 3 through the machine — class predicts each output aloud before it appears. When 0 gives 1, name it: 'that kills $+5$ dead — $0 + 5$ would be 5'. After each prediction, turn to a back-row pupil to say what the machine did, then revoice anyone who reasons 'the jumps get bigger so it's a times'.
4. **Work Out the Rule in Your Copy.** Pupils write both possible rules for each pair of Machine A, then test on the next pair before circling. Walk the room watching for anyone who circles after checking only the first pair — nudge them to try the second before committing.

5. **Class Challenge.** Brisk turns at the board — before each machine, ask 'which number would settle it fastest?' and let pupils choose 0 or 1. On the two-step machines hold out for both steps in order; 'times 2' alone is only half the rule. Do not skip the last machine for time — it's the one where +5 and $\times 4 + 2$ agree on the first pair and only split on the second.
6. **What Did We Notice?.** Ask which single test-input separates +5 from $\times 3 + 1$ fastest. Steer toward 0 and revoice: 'feeding in zero strips away the times part, so whatever comes out is the add part.' If a pupil offers 1, accept it and ask which is quicker to work out in your head.

COMMON MISCONCEPTIONS

⚠ Pupils lock onto +5 from the first pair (2→7) and stop — they never test it on 5→16, where it gives 10 instead of 16.

On the IWB, line the machine's outputs up under each input and show +5's predictions pulling away: 7 matches, but 10 vs 16 doesn't. Say 'a rule has to fit every pair, not just the first one' and make testing the second pair the habit.

⚠ On a two-step machine, a pupil says the rule is 'times 2' and thinks they're done — they've caught the multiply but missed the +3.

Feed their 'times 2' rule through on the board: 4→8, but the machine gave 5. Ask 'what do we do to 8 to land on 5?' — wait, no, land on the real output — so they hunt for the second step. Insist both steps are named in order.

⚠ Pupils treat the last Class Challenge machine as solved after one pair, because +5 and $\times 4 + 2$ both fit 1→6.

Send a second number through before anyone commits. When +5 and $\times 4 + 2$ disagree on the second pair, name it out loud: 'one pair can look like two different rules — that's exactly why we always try a second.'

DIFFERENTIATION

EMERGING

- Stay on single-step machines ($\times 4$, +7) for these pupils while the class moves to two-step. The 0→7 and 0→4 pairs hand the answer over with almost no arithmetic.
- In the copybook step, give them Machine A with the two candidate rules already written — their job is only to test each on the second pair and circle, not invent the rules.

DEVELOPING

- After Machine A, ask them to invent a hidden machine of their own for a two-step rule, write two pairs, and see if a neighbour-at-the-board later could crack it.
- Pose: 'a machine gives 3→11 and 5→17 — could it be a two-step rule with a subtract in it?' and have them find one.

PROFICIENT

- Give a machine where only one pair is shown (e.g. 4→13) and ask: how many different rules fit this single pair? Have them list three and explain why one pair can never be enough — a copybook moment, no board time needed.
- Ask them to explain to the teacher, in their own words, why 0 is such a powerful test-input and when it would fail to help (a machine with no add or subtract step).

✦ **Cross-curricular:** Tie to STE — a function machine mirrors a rule in a spreadsheet formula; pupils can describe a simple 'input $\times 3 + 1$ ' cell rule in words.

ANSWER KEY

a) +5

b) $\times 3 + 2$

c) $\times 5 + 1$

Q1: +10

Q2: $\times 3$

Q3: +6

Q4: $\times 4 + 1$

Q5: $\times 3 + 2$

a) **Never** : Many different rules can fit a single pair, so more pairs are needed.

b) **Always** : Two different rules always disagree on some input: $\times 2$ and +2 both send 2 to 4, but 3 goes to 6 and to 5.

c) Always : $2 \times 3 + 1 = 7$ and $5 \times 3 + 1 = 16$, so both pairs match.

d) Sometimes : $\times 3$ works, but $+8$ also works, so extra pairs are needed to decide.

EXTENSION SHEET · STRETCH ANSWERS

S1: $\times 2 + 4$

S2: $\times 6 + 4$

S3: $\times 6 + 6$

S4: $\times 3 + 7$

S5: $\times 5 + 1$