

## Lesson 82 SHAPE &amp; SPACE

# Construction: triangles from sides, bisecting lines and angles

## CURRICULUM ALIGNMENT

SHA.SHP.4a

construct 3-D and 2-D models or structures given defined measurements and/or specific conditions.

SHA.SHP.4b

investigate and construct angles in the context of shape; and solve angle-related problems.

## WHAT THIS LESSON TEACHES

To build a triangle with exact side lengths use only compass and ruler: draw one side then swing arcs set to the other two lengths so they cross at the third corner. Bisect a line or angle by swinging equal arcs from its ends or arms and joining the cross points.

→ Draw base 7 cm, swing 5 cm arc from one end and 6 cm arc from the other; join the crossing point.

→ Bisect a 70° angle; the two new angles are each 35°.

## MODEL THIS ON THE BOARD

## CONSTRUCT SIDES 5 CM, 7 CM, 6 CM

- 1 Draw a straight base exactly 7 cm long with the ruler.
- 2 Set the compass to 5 cm and swing a light arc from the left end of the base.
- 3 Set the compass to 6 cm and swing a light arc from the right end of the base.
- 4 Mark the point where the two arcs cross; that is the third vertex.
- 5 Join the third vertex to each end of the base with the ruler to finish the triangle.

## LESSON ARC

Open by asking whether a 5/7/6 cm triangle can be drawn exactly — hold the question, don't reveal the arc trick. Demo three constructions live: swing arcs from each end of a 7 cm base, then bisect a line and a 70° angle, holding out for the crossing points as the precision idea. Pupils build the 5/7/6 triangle at their desks, then a 6/8/10 in copybook. Class Challenge practises a scalene triangle and both bisections, protractor-checking the 45°.

## TEACHING MOVES

1. **Getting Started.** Take two or three hands-up ideas, then keep the question live: 'can we get all three sides right at the same time?' Resist the urge to show the arc trick — that's the discovery of the next step, and giving it away flattens the hook.
2. **Watch and Notice.** Build all three live on the board — don't narrate a script. On the triangle, ask 'why start with the longest side?' before swinging the 5 cm and 6 cm arcs. Each time an arc crosses, tap the crossing point and say 'that's the only spot where both lengths are right at once.'
3. **Try It Together.** Run the circle-tool interactive in explore mode so the on-screen arc-swing mirrors the paper. Bring one or two pupils up to show the set-plant-rotate hold before the class draws. Then circulate — the single most common slip is the compass point drifting mid-swing, so watch the plant-hand.
4. **Construct the Triangle in Your Copy.** Walk the room glancing for the base drawn first and arcs meeting cleanly — this is practice, not marking. If a pupil asks about the  $6^2+8^2=10^2$  note, flag it as a next-year Pythagoras teaser and move on; don't stop to prove it.

5. **Class Challenge.** Brisk practice — pupils take turns at the board, the class confirms each construction before moving on, and every pupil builds all three on paper. On the  $90^\circ$ -bisection, have pupils lay the protractor on their own halves to confirm  $45^\circ$  — that's where the compass-precision claim gets tested against a measuring tool.
6. **What Did We Notice?.** Listen for pupils naming the fixed compass distance as the reason the crossing is exact. Revoice it: 'so the arc holds the length steady for us — our eyes can't do that.' Push back gently on 'close enough by eye' — a millimetre out on each side and the triangle never closes.

### COMMON MISCONCEPTIONS

⚠ Pupils let the compass width creep — they re-open it slightly between planting and swinging, so the arc no longer represents 5 cm and the crossing point lands wrong.

Stop and demonstrate the set-plant-rotate hold on the board: set the width against the ruler, plant the point, and swing only the pencil-arm. Have the pupil re-measure their arc radius against the ruler — if it's not 5 cm, the width moved.

⚠ Pupils draw the two arcs so faintly or so short that they never actually cross, then guess where the third corner 'should' go.

Show that each arc needs to sweep well past where you expect the crossing. Redraw one long, generous arc on the board so pupils see the full curve, then have them extend their own arcs until they genuinely intersect — no guessing the vertex.

⚠ After bisecting the  $90^\circ$  angle, pupils are surprised the protractor reads  $45^\circ$  and half-suspect the compass 'got lucky'.

Frame this as the point of the investigation, not luck. Have several pupils read their own halves aloud — they all land on  $45^\circ$ . The equal arcs force equal halves every time; the protractor is confirming, not correcting.

### DIFFERENTIATION

#### EMERGING

- Pre-draw the 7 cm base lightly in the copybook so the pupil only has to set the compass and swing the two arcs, not manage the whole figure at once.
- Let these pupils work the string-and-pencil fallback first — the bigger swing radius makes the fixed-distance idea physically obvious before they move to the compass.

#### DEVELOPING

- After the 6/8/10 triangle, give a set that won't close — sides 3 cm, 4 cm, 9 cm — and ask them to try, then say what went wrong (the short arcs never reach each other).
- Ask them to bisect their bisector — cut the 14 cm line into four equal 3.5 cm parts using arcs only, no measuring the quarters.

#### PROFICIENT

- Construct an equilateral triangle with side 8 cm, predict all three angles are  $60^\circ$ , and be ready to explain from the equal arcs why the angles must be equal — narrate this at the board for the class while others finish.
- Pose: using only the bisecting method twice, can you construct a  $45^\circ$  angle from nothing but a straight line? Let them investigate in the copybook without being shown.

✂ **Cross-curricular:** Tie to visual arts — Celtic knotwork and rose windows are built from compass arcs; pupils swing overlapping arcs to make a six-petal 'daisy' design.

### ANSWER KEY

**Q1:** Construct triangle with sides 4 cm, 5 cm, 6 cm

**Q2:** triangle 9 cm, 11 cm, 13 cm

**Q3:** two  $35^\circ$  angles

**Q4:** triangle 12 cm, 16 cm, 20 cm (right-angled)

**Q5:** triangle 14 cm, 15 cm, 17 cm

**Q6:** equilateral  $\triangle$  side 4 cm

**a) Always :** The arcs of 5 cm and 6 cm must cross at the unique third vertex.

**b) Always :** An angle bisector cuts the original angle into two equal halves.

**c) Never :** Compass arcs alone locate the third vertex; no angle measure is needed.

d) Sometimes : True for 4 cm, 5 cm and 6 cm; false for 2 cm, 3 cm and 8 cm, where the short sides cannot meet.

#### EXTENSION SHEET · STRETCH ANSWERS

S1: isosceles

S2: a regular hexagon

S3: a rectangle

S4:  $39^\circ$

S5: an isosceles triangle