

The mean (average) of a small set: a gentle first look

CURRICULUM ALIGNMENT

DAT.DAT.3

pose questions of interest and collect, display and critically analyse data in a range of ways for a range of purposes and communicate the findings.

WHAT THIS LESSON TEACHES

The mean is the fair, even share. Gather every amount together into one total, then share that total out equally so each one has the same. Add first, then share.

→ $3 + 5 + 2 + 6 = 16$, then 16 shared between 4 friends gives a mean of 4 cubes each.

→ Four towers of 3, 5, 2 and 6 level out to 4 each.

MODEL THIS ON THE BOARD

THE MEAN OF 3, 5, 2 AND 6

- 1 Add the four amounts: $3 + 5 + 2 + 6 = 16$.
- 2 Share the total fairly between the 4 amounts: $16 \div 4 = 4$.
- 3 Each share is 4, so the mean is 4.

16			
4	4	4	4

DISCUSSION STARTER

REFLECT

Did thinking of the mean as a fair share help you find it? What still feels tricky?

LESSON ARC

Stand four real cube towers (3, 5, 2, 6) at the front and take guesses before any counting. Level them live — tallest gives to shortest until all four sit at 4 — then write the same as $3+5+2+6=16$, shared by 4. The pivot is that levelling and add-then-share reach the same answer. Pupils build and total three towers in their copy, then the Class Challenge practises add-then-share, including a euro set.

TEACHING MOVES

1. **Getting Started.** Stand the four real towers (3, 5, 2, 6) where all can see. Five seconds' quiet think-time, then take three hands-up guesses. Hold the answer back — don't confirm anything until the levelling in the next step lands it.
2. **Watch and Notice.** Level the towers by hand, narrating 'the tallest gives to the shortest' as you move cubes onto the 2-tower and up. Then write $3+5+2+6=16$, 16 shared by 4 is 4. Don't announce that both roads reach 4 — wait for a pupil to spot it.
3. **Try It Together.** Reveal one set at a time, never the whole list. Give 4,4,4,8 a full board build by a pupil; run the other three as quick verbal levelling. Watch for a cube quietly lost or a tower pushed too tall — re-count one tower to check. Pause on 4,4,4,8: the mean (5) is no tower's height, which surprises them.
4. **Watch the Working.** Walk the annotated working line by line, pointing at each piece. Stress the one move pupils get wrong: you share by how many towers there are, not by one of the heights.
5. **Work It in Your Copy.** Give three evenly-sharing heights (e.g. 2, 4, 6). Pupils draw the towers, write the total, then the fair share underneath. Glance for the two slips: adding correctly but forgetting to share, or sharing by the wrong number of towers. This is practice, not marking.

6. **Class Challenge.** Brisk practice round — pupils take turns at the board, the class predicts each fair share before it's confirmed, then move on. Give the 4,4,4,4 set a beat: an already-even set has that same value as its mean, nothing moves.
7. **What Did We Notice?.** Ask why levelling matches add-then-share. Listen for 'moving cubes never adds or loses any' and revoice it: whether we push cubes or add and share, we split the same total into equal parts.

COMMON MISCONCEPTIONS

⚠ A pupil finds the mean of 4, 4, 4, 8 and says '4, because that's the height most of the towers are' — they expect the mean to be one of the actual values.

Build 4, 4, 4, 8 live and level it: cubes move off the 8 until all four sit at 5, a height none of them started at. Say 'the fair share doesn't have to match any one tower.' The 5,5,5,5,10 set makes the same point.

⚠ Pupils add the heights correctly to get 16 but then stop, or share by one of the heights instead of by the number of towers (e.g. $16 \div 6$).

Point back to the annotated working from 'Watch the Working' and ask 'how many towers are we sharing between?' Count the towers aloud together, then divide by that count. The number of towers, not any height, is the divisor.

⚠ When levelling, a pupil loses a cube (or adds a spare) so the towers don't all come out equal.

Re-count one tower against another and total the whole set before and after — if the total changed, a cube went missing. Rebuild from the original heights; the total must stay the same for the share to be fair.

DIFFERENTIATION

EMERGING

- Stay with three-tower sets that share evenly (2, 4, 6) while the class moves to four- and five-tower sets. Pupils level the real cubes by hand before writing anything.
- Use the printed cube-tower strips so the pupil shades and counts squares rather than balancing loose cubes — the levelling reads the same.

DEVELOPING

- After the copybook page, hand a set that doesn't share evenly (2, 3, 4) and ask what happens — a real mean between whole numbers, worth a discussion.
- Pose a missing-height puzzle: three towers are 3, 5 and ?, and the fair share is 5. How tall is the third tower?

PROFICIENT

- Narrate a harder Class Challenge variant from the front: give a five-value set with a large outlier (2, 2, 2, 2, 12) and ask them to predict the mean before building, then explain why one tall tower pulls the fair share up.
- Ask them to invent a four-tower set whose mean is 6 but where no tower is 6 high, and justify why their set works.

✓ **Cross-curricular:** Tie to PE — after a beanbag-throw round, pupils pool each group's scores and find the mean throw distance for the group.

ANSWER KEY

Scaffold: visual check on the page

Q1: 4

Q2: 3

Q3: 5

Q4: 6

Spot: Aoife divided by 3 instead of by how many numbers there are. There are **4** numbers, so $16 \div 4 = 4$.

EXTENSION SHEET · STRETCH ANSWERS

S1: 14

S2: 11

S3: 11

S4: 8

S5: 15

Investigation: Make the mean — open-ended; scan pupils' working for valid solution paths rather than a single answer.