

Volume: counting unit cubes in a cuboid

CURRICULUM ALIGNMENT

MEA.MSR.3a

compare, estimate and measure length, weight, capacity, area and volume using appropriate instruments and record and communicate appropriately.

NUM.OPS.3

understand and apply flexibly the four operations; and the relationships between operations.

INTERACTIVES **Volume Builder** · challenge, display, explore

WHAT THIS LESSON TEACHES

The volume of a cuboid is how many equal unit cubes fill it with no gaps. Find how many cubes fit in the bottom layer, then multiply by the number of layers stacked on top.

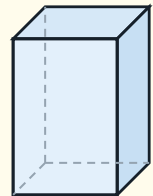
→ A box packed with 12 unit cubes has a volume of 12 cubic units.

→ If the bottom layer holds 6 cubes and there are 2 layers, the box holds 12 cubes.

MODEL THIS ON THE BOARD

COUNTING CUBES IN A CUBOID

- ① The bottom layer is **2** cubes wide and **2** cubes deep, so it holds $2 \times 2 = 4$ cubes.
- ② The cuboid is **3** cubes high, so there are **3** equal layers.
- ③ Multiply: $4 \times 3 = 12$.



DISCUSSION STARTER

REFLECT How is counting cubes for volume different from counting squares for area?

LESSON ARC

Hold a real unit cube against a small box and take three estimate guesses — how many cubes fill it? On the volume-builder interactive, fill a 3-by-2 base (6 cubes), pause and predict before the second layer lands (12). Pupils build cuboids at the board layer by layer in Try It Together, then sketch a $2 \times 2 \times 3$ cuboid in their copy and label the cube count. Class Challenge is the checked practice bank, ending on a build-12-your-own-way target.

TEACHING MOVES

1. **Getting Started.** Hold one unit cube against a matchbox or cube-tray and take exactly three hands-up guesses — you want a spread, not a chorus. Write the three estimates in a corner of the board so you can circle the closest at the end. Reveal no number yet: the whole lesson answers 'how would we know for sure?'
2. **Watch and Notice.** Fill the 3-by-2 base one cube at a time — count '3 across, and 3 again, six'. Then STOP and ask for a prediction before the second layer appears: this pause is the make-or-break moment. Revoice a strong prediction — 'one layer is six, so two layers is six and six more' — and name each cube as one cubic unit here.

3. **Try It Together.** Pupils set width, depth and height at the board and read the count for the class to agree or correct aloud — no marking this round. Insist on the two-step count every time: 'how many in one layer? how many layers?' Don't let anyone slide back to counting cube-by-cube once the $4 \times 3 \times 1$ base is built.
4. **Sketch the Cuboid in Your Copy.** Pupils sketch a 2-wide, 2-deep, 3-high cuboid in copy and write the total. Walk the room checking the base is 2 by 2 and there are three layers stacked. For anyone stuck, prompt only with 'how many in one layer, and how many layers?' — don't hand them 12.
5. **Class Challenge.** Brisk turns at the board: build, press Check, class thumbs-up to confirm, clear, next. Save maths-talk for the final open target — 'who made 12 a different way?' — since $2 \times 6 \times 1$ and $3 \times 2 \times 2$ both check as correct. Watch the stepper order is width, height, depth to match the labels.
6. **What Did We Notice?.** Ask how you'd find the whole cuboid without counting every cube. Listen for 'add the layer on again each time' or 'times the layer by how many layers', then revoice the strongest: 'so the layer times the height gives the whole box.'

COMMON MISCONCEPTIONS

⚠ Pupils go back to counting every single cube one by one, even on the $3 \times 3 \times 3$ build, ignoring the layer shortcut.

Freeze the board mid-count and cover the top two layers with your hand. 'How many in just this bottom layer? Now — how many identical layers?' Rebuild the count as 9 and 9 and 9 so they hear the repeat, not 27 separate cubes.

⚠ When a second layer lands, pupils predict the base number again (e.g. still 6 for two layers) rather than adding another full layer.

Before revealing the second layer on the volume-builder, hold the prediction and point at the empty space above the base. 'The same six cubes are coming again — how many altogether?' Confirm by counting the second layer aloud on top.

DIFFERENTIATION

EMERGING

- Keep these pupils on single-layer, height-1 builds on the interactive ($4 \times 3 \times 1$, $5 \times 2 \times 1$) until the base-count is secure before adding a second layer.
- For the copy sketch, let them draw only the flat base as a 2-by-2 grid first, count 4, then add the '×3 layers' as a written note rather than drawing all three tiers.

DEVELOPING

- After the copy sketch, ask them to draw a second cuboid with the SAME 12 cubes arranged differently and check both totals still land on 12.
- Give the missing-layer variant: 'a base of 5 cubes, total is 20 — how many layers?' before they reach the open Class Challenge target.

PROFICIENT

- Pose: 'a cuboid holds 24 cubes — list every different width-by-depth-by-height build you can find, and say how you know you have them all.' Pair with a copy moment to record the systematic list.

✓ **Cross-curricular:** Tie to Geography — count how many unit cubes fill a real matchbox or lunchbox, then compare which classroom container holds the most cubes.

ANSWER KEY

a) 5, 4, 3

b) 20

c) 60 cubic units

Q1: 21 cubes

Q2: 12 cubes

Q3: 60 cubes

Spot: Sean counted only one layer and forgot the height. There are 2 layers of 12 cubes, so volume = $4 \times 3 \times 2 = 24$ cubes.

EXTENSION SHEET · STRETCH ANSWERS

S1: 30 cm^3

S2: 18 cm^3

S3: 72 cm^3

S4: 108 cm^3

S5: 75 cm^3

Investigation: Pack the box — open-ended; scan pupils' working for valid solution paths rather than a single answer.