

Same perimeter, different area: an investigation

CURRICULUM ALIGNMENT

SHA.SHP.3b

represent shapes with drawings and models, and calculate dimensions of shapes.

ALG.PRR.3a

identify rules that describe the structure of a pattern and use these rules to make predictions.

INTERACTIVES

Shape Measurer · display

WHAT THIS LESSON TEACHES

The perimeter is the total distance all the way around the outside of a flat shape, found by adding the length of every side. Two rectangles can share the same perimeter yet cover a different number of squares inside.

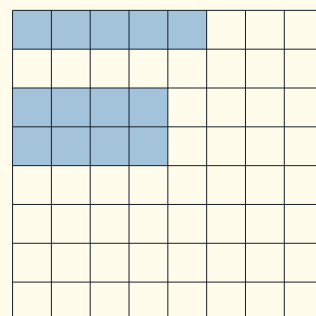
→ A 4 cm by 2 cm rectangle has a perimeter of $4 + 2 + 4 + 2 = 12$ cm and covers 8 cm^2 .

→ A 1 by 5 and a 2 by 4 rectangle both have perimeter 12 cm, but the 2 by 4 holds 8 cm^2 while the 1 by 5 holds only 5 cm^2 .

MODEL THIS ON THE BOARD

SAME FENCE, DIFFERENT SPACE

- ① Both rectangles use the same fence of **12 cm**: $1 + 5 + 1 + 5 = 12$ and $2 + 4 + 2 + 4 = 12$.
- ② The long thin rectangle is **1** row of **5** squares, so its area is **5 cm^2** .
- ③ The fatter rectangle is **2** rows of **4** squares, so its area is **8 cm^2** .
- ④ Same perimeter, yet **8 cm^2** holds more space than **5 cm^2** .



DISCUSSION STARTER

REFLECT

What surprised you most about shapes that share a perimeter but have different areas?

LESSON ARC

Put two same-perimeter rectangles (1×5 and 2×4 , both 12 cm) on the IWB and get three hands-up predictions to the fence-versus-space question — then leave it open. In Watch and Notice, build 1×5 , 2×4 , 3×3 , reading areas off the tool into a board table so the pattern lands at the square. Pupils draw two perimeter-10 rectangles in copy. The perimeter-20 hunt is the Student Activity Book page that follows.

TEACHING MOVES

1. **Getting Started.** Point at both rectangles and trace a finger round each edge so pupils see the equal fence before you ask the question. Take exactly three hands-up guesses, write both 'yes' and 'no' on the board, and resist confirming — that split is the tension you'll resolve at the wrap.
2. **Watch and Notice.** Ask for a thumbs prediction before each reveal, then read the area off the tool and log it in a board table (5, 8, 9). Don't name the rule up front — after the third rectangle ask 'what's

happening to the space inside?' and revoice a pupil's words. Pause hard on the 3×3 square; that's the pay-off.

3. **Draw Two in Your Copy.** Walk the room while pupils draw two different perimeter-10 rectangles. Don't mark — where a shape is wrong, just prompt 'does your fence add to ten?' Watch that the two shapes are genuinely different (1×4 and 2×3 both qualify).
4. **Try It Together.** Bring individual pupils up to build $1 \times 7 \rightarrow 2 \times 6 \rightarrow 3 \times 5 \rightarrow 4 \times 4$ in that strict order, class thumbs-predicting each area before it's read off. Keep repeating 'the fence is always 16 cm'. When someone predicts the 1×7 too high, welcome it — that 'long fence means lots of space' instinct is exactly what today overturns.
5. **Class Challenge.** Reveal the three jobs one at a time. Pupils take turns building 1×5 , 2×4 , 3×3 on the tool; class checks each area and you log it. Keep it brisk. When someone offers 5×1 as new, ask the class — agree it's 1×5 turned round. Close by revoicing that the square wins.
6. **What Did We Notice?.** Return to the two opening guesses and ask which held up. Listen for 'closest to a square holds the most'. If a pupil still feels longer fence ought to mean more area, point back at the board table where one perimeter gave three different areas.

COMMON MISCONCEPTIONS

⚠ Pupils treat perimeter and area as the same measure — 'the fence is longer so it holds more', predicting the 1×7 rectangle to be the biggest.

Point to the edge for perimeter and the inside squares for area on the same rectangle. Build the 1×7 and the 4×4 side by side on the tool: identical fence, but count the squares inside each — 7 against 16.

⚠ Pupils list 5×1 and 1×5 as two separate rectangles when hunting for every rectangle with a given perimeter.

Drag one rectangle on the tool and rotate it a quarter-turn so the class sees the same shape lying down. Agree together that a rectangle turned round is not a new rectangle, and cross the duplicate off the board list.

DIFFERENTIATION

EMERGING

- Give the copybook pair a start: pre-draw the 1×4 rectangle on squared paper so they only need to find and draw one more perimeter-10 shape and count its squares.
- During the perimeter-12 hunt, let these pupils count the squares inside each built rectangle one at a time rather than track the whole list.

DEVELOPING

- After the perimeter-12 list, ask them to try perimeter 14 and predict which rectangle will win before checking on the tool.
- Add a justification step in the copy: under their circled rectangle, write one sentence saying why it holds more even though the fence matches.

PROFICIENT

- While the class finishes the board list, pose: 'if we allowed half-squares, could you make a rectangle with perimeter 12 that holds even more than 9 cm^2 ? What would it look like?' — a teacher-narrated stretch toward the square-is-best idea, or pull them ahead onto the perimeter-20 Student Activity Book page.

✓ **Cross-curricular:** Tie to Geography — pupils sketch two fields with the same length of stone wall around them and decide which holds more grazing land.

ANSWER KEY

b) 3 by 9: 27 cm^2 ; 6 by 6: 36 cm^2 (the page's 5 cm^2 and 8 cm^2 fit a 1 by 5 and a 2 by 4)

c) 1 by 4: 4 cm^2 ; 2 by 3: 6 cm^2 ; circle the 2 by 3

Q1: 16 cm

a) **Sometimes** : Same-perimeter rectangles can differ in area (1 by 5 and 2 by 4 both have perimeter 12 cm yet areas 5 cm^2 and 8 cm^2) or match if congruent, so only sometimes.

b) **Never** : Perimeter is the total length around the shape and is measured in cm; square centimetres measure area.

c) **Never** : Perimeter is found by adding every side (e.g. $4+2+4+2$); multiplying length by width gives area.

d) **Always** : Every rectangle's area is length times width, matching the lesson examples ($4 \times 2 = 8$, $2 \times 4 = 8$, $1 \times 5 = 5$).

EXTENSION SHEET · STRETCH ANSWERS

S1: 30 cm^2

S2: 10 cm^2

S3: 38 m

S4: 38 m

S5: 48 m

Investigation: Fence the biggest garden — open-ended; scan pupils' working for valid solution paths rather than a single answer.