

Generalising number patterns and sequences

CURRICULUM ALIGNMENT

ALG.PRR.4a

identify, explain and apply generalisations, including properties of operations, mathematical models and patterns.

ALG.PRR.4b

represent mathematical structures in multiple ways, including verbal expressions, diagrams and symbolic representations.

INTERACTIVES Function Machine · challenge, display, explore

WHAT THIS LESSON TEACHES

A number pattern has a hidden rule that turns each position number into the next term, often a multiply step then add or subtract.

→ In 4, 7, 10, 13 the rule is position times 3 then add 1.

→ For 6, 11, 16, 21 the rule is position times 5 then add 1.

MODEL THIS ON THE BOARD

FIND THE RULE FOR 4, 7, 10, 13

- 1 Look at the first term: position 1 gives 4.
- 2 Try multiply by 3 then add 1: 1 times 3 is 3, plus 1 is 4.
- 3 Check next: 2 times 3 is 6, plus 1 is 7.
- 4 The rule works for every term.

LESSON ARC

Open with 4, 7, 10, 13... and ask for the 100th term — let pupils feel how slow listing would be. Then build the function-machine table for Machine A on the IWB, lining positions 1, 2, 3 under outputs 4, 7, 10 so pupils discover $\times 3$ then $+1$ themselves. Pupils note the step and rule for three sequences in their copy. The Class Challenge works five hidden-rule machines at the board, and the wrap loops back to answer the 100th-term question (301).

TEACHING MOVES

1. **Getting Started.** Take three quick estimates of the 100th term — not open call-outs — and resist showing any method. Leave 'could we find it without writing all 100?' hanging as the hook; the slow-listing frustration is the point.
2. **Watch and Notice.** Build Machine A fully on the input-output table: line positions 1, 2, 3 under outputs 4, 7, 10 and draw out 'three lots of the position, plus 1 left over each time'. Name the rule $\times 3$ then $+1$ only after the class has said it — do not announce it first.
3. **Try It Together.** One pupil feeds inputs at the board while the watching class says the step aloud in unison. Insist they state the step before the rule ('up in 5s, so there's a $\times 5$ '). Re-voice a strong move: 'so you used two pairs to be sure, not just one.'
4. **Note the Step and the Rule in Your Copy.** Walk the room glancing for three things: position numbers under each term, the circled step, and a rule written in words not just numbers. Prompt anyone stuck with 'what number is going in to make this term come out?'

5. **Class Challenge.** Run the five machines briskly — one pupil at the board, class confirms, move on; don't re-explain each. Watch the 7, 17, 27, 37 machine: it takes away ($\times 10$ then -3), so catch pupils trying to add. Fast finishers mouth the next output rather than working ahead.
6. **What Did We Notice?.** Write 3, 8, 15, 24 and mark the jumps $+5$, $+7$, $+9$ so pupils see the step itself growing. Re-voice: 'when the step holds still, one rule works for every term; when the step changes, we think differently.' Display-only discussion — nothing written.

COMMON MISCONCEPTIONS

⚠ Pupils nail the step ($+3$) but write the rule as just ' $+3$ ' or 'add 3', skipping the multiply — so they can't jump to the 100th term.

Back at the table, ask 'what's at position 1? what's at position 100?' and show that adding 3 each time still means counting up. Line three lots of the position under each output so the $\times 3$ becomes visible, then the $+1$ lands it.

⚠ On the 7, 17, 27, 37 machine, pupils force an add and get stuck — they write $\times 10$ then $+3$ and the outputs don't match.

Have them test position 1: $\times 10$ gives 10, but the output is 7, so we went down by 3. Re-voice that the multiply can overshoot, and then we take away to land it.

DIFFERENTIATION

EMERGING

- Give these pupils the position numbers already written under each term in the copybook task, so they only hunt the step and rule, not the structure.
- Stay with the $+2$ and $+3$ machines where the multiply matches a familiar table; the trickier $\times 6$ and $\times 10$ machines are teacher-supported at the board.

DEVELOPING

- After the copybook sequences, hand a single pair (e.g. $4 \rightarrow 23$) and ask which rules could fit, then a second pair to pin it down.
- Ask for the 50th term of one of the Class Challenge machines, applying the rule directly rather than extending the list.

PROFICIENT

- During the Class Challenge wait, narrate a harder variant: 'find a machine where position 1 gives 0 — what's the rule?' and have them justify it.
- Pose the changing-step 3, 8, 15, 24 early: can a single multiply-and-add rule ever fit it? Ask them to explain why not before the class reaches the maths-talk.

✂ **Cross-curricular:** Link to Geography — read bus or DART timetables where departures repeat at a constant step and predict a later time using the rule.

ANSWER KEY

Q1: 16

Q2: 31

Q3: add 3; 26

Q4: add 10; 97

Q5: $n(n+2)$; 63

a) **Always** — A constant step means the rule is the same for every position.

b) **Always** — The rule position times 3 plus 1 gives the exact term no matter how far.

c) **Never** — This pattern has changing steps so no single multiply-and-add rule fits all.

d) **Sometimes** — It works if the rule is found first, but not if the steps keep changing.

EXTENSION SHEET · STRETCH ANSWERS

S1: $\times 6 + 4$

S2: $\times 3 + 8$

S3: $\times 3 + 6$

S4: 14

S5: 2

Investigation: Growing patterns — open-ended; scan pupils' working for valid solution paths rather than a single answer.