Lesson 79 SHAPE & SPACE

Properties and faces, edges, vertices of polyhedra

CURRICULUM ALIGNMENT

SHA.SHP.4a construct 3-D and 2-D models or structures given defined measurements and/or specific conditions.

ALG.PRR.4a identify, explain and apply generalisations, including properties of operations, mathematical models and patterns.

INTERACTIVES 3D Shape Inspector · challenge, display, explore

WHAT THIS LESSON TEACHES

A flat surface of a solid shape is a face. A cube has six square faces, twelve edges and eight vertices.

→ A cardboard box has six faces.

→ A cube has twelve edges and eight vertices.

MODEL THIS ON THE BOARD

COUNTING THE CUBE

1. Count the flat faces on all sides of the cube: front back left right top bottom gives **6**.
2. Count where two faces meet to find the edges: **12** in total.
3. Count the pointed corners for the vertices: **8** altogether.
4. Check the pattern with **6 + 8 - 12 = 2**.

LESSON ARC

Open with the shape inspector cube on the IWB and take hands-up guesses for the corners before counting — pupils forget the hidden ones. Turn each of the four solids so the class counts faces, edges and vertices including the back. Pupils rule a four-row table in their copy. Then the whole class works F + V − E for each solid, predicting before each check, until a pupil says it's always 2 — Euler's rule, found not told.

TEACHING MOVES

1. **Getting Started.** Show the cube still, give five seconds of quiet think-time, then take three hands-up guesses for the corners — don't confirm any. Expect '4' or '6' from pupils counting only the front; let that wrong guess sit so the hidden-corner problem is theirs to solve.
2. **Watch and Notice.** Turn each solid fully so pupils catch the back edges and the two round-the-back vertices classes always lose. On the cuboid, ask 'why do the numbers match the cube even though it looks different?' Do not point out any number pattern yet — that noticing is the pupils' job in steps 4 and 5.
3. **Build the Table in Your Copy.** Pupils rule four columns — Solid, Faces, Edges, Vertices — and one row per solid. Walk the room glancing at headings and counts; no marking. Bite your tongue on the F + V − E pattern even as pupils start to notice their rows behaving alike — that noticing fuels the next step.
4. **Try It Together.** One pupil comes to the board, turns the solid and taps each part once so nothing is double-counted. Before each solid, take two hands-up face-count guesses to keep the back rows in it. Insist the board pupil turns the solid right round before claiming a total — the far-side edges are where counts go wrong.

5. **Class Challenge.** Brisk board work: pupils predict F + V − E, then confirm the counts and work it in their copy, one solid at a time. Keep it moving — don't re-explain the counting. Hold out for a pupil to say 'it's 2 every time' rather than announcing it, then ask 'would you bet it works for a solid we haven't tried?'
6. **What Did We Notice?.** Push pupils from 'it just does' to a reason or prediction. Revoice a strong claim as a conjecture: 'so you're saying it works for every solid, not just these four.' Ask what would have to happen for it to fail. Name Euler's rule now they've found it.

COMMON MISCONCEPTIONS

⚠ Pupils count only the faces, edges and corners pointing at them — 'a cube has 4 corners' — because the far side is out of sight.

Make the pupil at the board turn the solid a full turn before they give a total, tapping each part once. Revoice: 'so we turned it right round to catch the back corners.' The round-the-back count is the whole skill today.

⚠ On the triangular prism, pupils miscount faces — they see three rectangles and forget the two triangular ends, or count the ends as one.

Turn the prism to face each triangular end in turn on the shape inspector. Count aloud as 'two triangles plus three rectangles — five faces.' Have a pupil tap all five before the class agrees the total.

⚠ In the maths-talk, pupils treat the always-2 result as a coincidence — 'it just came out the same each time.'

Ask 'what would have to happen for F + V − E NOT to equal 2?' Reframing it as a claim that could fail pushes pupils from noticing toward a conjecture, which is the point of the investigation.

DIFFERENTIATION

EMERGING

- Give these pupils the cube and cuboid only for their copy table — the two solids with matching, memorable counts — and support the count at the teacher table.
- Have them tap and count on the shape inspector one part-type at a time (all faces, then all edges, then all vertices) rather than all three at once.

DEVELOPING

- Once their table is done, hand them a solid they didn't count — a pentagonal prism on the inspector — and ask them to predict F + V − E before counting.
- Ask them to write, under their table, why the cube and cuboid share the same three numbers even though one is stretched.

PROFICIENT

- Pose: 'invent a solid on paper and work out its F, V and E — can you make one where F + V − E is NOT 2?' Let them hunt; the near-miss cases are where the reasoning bites.
- Ask them to explain Euler's rule to a pupil who only counted the cube, using their own copy table as the evidence.

↗ **Cross-curricular:** Tie to visual art — pupils sketch one solid from two viewpoints so the hidden edges show as dashed lines, the way designers draw.

ANSWER KEY

Scaffold: visual check on the page

Q1: 8

Q2: 12

Q3: 5

a) Always — Every cube has exactly six faces.

b) Sometimes — Some solids like pyramids have 5 faces.

c) Never — Edges and faces are different counts.

d) Always — Euler's rule holds for these solids.

EXTENSION SHEET · STRETCH ANSWERS

S1: a square-based pyramid

S2: a cuboid