Lesson 76 SHAPE & SPACE

Angle sum of a quadrilateral

CURRICULUM ALIGNMENT

SHA.SHP.4b investigate and construct angles in the context of shape; and solve angle-related problems.

ALG.PRR.4a identify, explain and apply generalisations, including properties of operations, mathematical models and patterns.

INTERACTIVES Polygon Angles · challenge, display, explore

WHAT THIS LESSON TEACHES

The four interior angles of any quadrilateral always add to 360 degrees.

→ 90 + 90 + 90 + 90 = 360

→ 100 + 80 + 100 + 80 = 360

MODEL THIS ON THE BOARD

FINDING THE MISSING ANGLE

1. Split the quadrilateral into two triangles by drawing one diagonal.
2. Each triangle sums to 180 degrees so together they make 2 x 180 = 360 degrees.
3. Add the three known angles then subtract from 360 to find the fourth.

LESSON ARC

Open problem-first: sketch a quadrilateral with one diagonal ruled across it and ask pupils to predict the angle sum from what they know about triangles — write predictions up unconfirmed. Move through three snapshot shapes, saving the irregular one to clinch that the total stays 360°. Pupils rule their own diagonal in the copybook and label 180° in each triangle. The drag-corner interactive shows angles trading off while the sum holds, then the Class Challenge works four missing-angle problems on the board.

TEACHING MOVES

1. **Getting Started.** Give five seconds of silent think-time before hands go up, then take three predictions for the four-angle total and write all three on the board without confirming any. Listen for 'two triangles, so two lots of 180' — hold that pupil's reasoning to pull back out at the reveal, don't validate it yet.
2. **Watch and Notice.** Point at each of the three snapshots in turn rather than pre-explaining. Let the class total the two right-angled shapes in their heads; then land on the irregular third one — nobody can guess it, yet it still reads 360°. Only now name the rule, quoting back the hook pupil: one diagonal, two triangles, 180 each, so 360.
3. **Split and Show It in Your Copy.** Walk the room checking the diagonal genuinely reaches an opposite corner — some pupils clip off a corner instead. This beat is construction only: they rule accurately with a ruler, then label 180° in each triangle and write 2 × 180° = 360°. No missing-angle sums here.
4. **Try It Together.** Bring individual pupils up to drag a corner while the rest predict which angle will shrink before the drag. The move that lands it: stretch one angle huge and watch another shrink as the sum readout holds at 360°. Revoice: 'so the angles trade off, but the pot is always 360.'
5. **Class Challenge.** Brisk practice — roughly two minutes per problem, four board turns inside the slot, class confirms each answer before moving on. Cue the method every time: 'add the three you know, then

take that from 360.' On the fourth problem watch for pupils who add 85 + 95 + 130 wrongly rather than misusing the method.

6. **What Did We Notice?.** Ask why two triangles guarantees 360°, and revoice a strong answer into one clean sentence so the whole class hears the reasoning chain. For the pentagon predict-forward, take a couple of guesses and leave it open — don't confirm the 540°.

COMMON MISCONCEPTIONS

⚠ A pupil rules a line that clips off one corner rather than joining two opposite corners, so they get a triangle plus a smaller quadrilateral instead of two triangles.

Catch it at the copybook step. Put your finger on the two corners that must be joined and have the pupil re-rule from corner to opposite corner. Ask 'how many triangles now?' — the two-triangle split is what makes the 2 × 180 argument work.

⚠ In the Class Challenge a pupil gets the fourth angle wrong not because of the method but because they add the three known angles incorrectly — e.g. 85 + 95 + 130 comes out short.

Separate the two steps out loud: 'the method is right — let's just re-add the three we know.' Redo the addition column-wise on the board, then subtract from 360. This keeps their confidence in the 360° rule intact while fixing the arithmetic slip.

⚠ A pupil thinks the 360° total only works for the neat right-angled shapes and that the irregular quadrilateral must be different.

Go back to the drag-corner interactive and stretch one angle right out while the sum readout stays on 360°. The shape bends wildly but the pot never changes — 'the angles trade off, the total holds.'

DIFFERENTIATION

EMERGING

- Let these pupils copy a ready-drawn quadrilateral with its diagonal already dotted in, so they only rule over the line and label 180° in each triangle rather than inventing the shape.
- In the Class Challenge, hand them the first problem (three 90s) — the square total is one they can reason without heavy arithmetic.

DEVELOPING

- After the copybook step, ask them to draw a second quadrilateral and rule the diagonal from the other pair of corners — does the two-triangle split still give 360°?
- Give a missing-angle problem where two angles are given and two are equal (e.g. 110°, 130°, and two matching angles) — they must halve the remainder.

PROFICIENT

- Pose the pentagon question early: split a five-sided shape from one corner, count the triangles, and predict the angle sum — then justify why it's 3 × 180°. Pair this with a copybook sketch.
- Ask them to write, in their own words for a younger pupil, why any four-sided shape totals 360° no matter how squashed it looks.

↗ **Cross-curricular:** Tie to visual arts — pupils sketch a tiling pattern of quadrilaterals and check the four angles meeting at a point relate to the 360° around a full turn.

ANSWER KEY

a) 360°

b) 360°

c) 360°

Q1: 30°

Q2: 100°

Q3: 50°

Q4: 125°

Q5: 75°

a) Always — This is the rule for any four-sided shape.

b) Never — They always add to exactly 360 degrees.

c) Always — Two triangles each sum to 180 degrees.

d) Never — Only special ones like squares and rectangles do.

EXTENSION SHEET · STRETCH ANSWERS

S1: 11°

S2: 99°

S3: $37°$

S4: $30°$

S5: $106°$

Investigation: Angle sums of polygons — open-ended; scan pupils' working for valid solution paths rather than a single answer.