Lesson 106 ALGEBRA

Properties of operations and the distributive law

CURRICULUM ALIGNMENT

ALG.PRR.4a identify, explain and apply generalisations, including properties of operations, mathematical models and patterns.

ALG.PRR.4b represent mathematical structures in multiple ways, including verbal expressions, diagrams and symbolic representations.

NUM.OPS.4 build upon, select and make use of a range of operation strategies.

INTERACTIVES Area Model · challenge, display, explore

WHAT THIS LESSON TEACHES

Operations obey **properties**: order doesn't matter for + and × (commutative), and the **distributive law** splits a product over a sum.

→ 6 × 23 = 6 × 20 + 6 × 3 = 120 + 18 = **138**.

	× 20	× 3
6	120	18

Total: 120 + 18 =

LESSON ARC

Open with 6 × 13 as a head-maths challenge — pupils try splitting the 13 before any rule is named. Then bring up the area-model rectangle and split 13 into a 10-piece and a 3-piece, naming the rule only after three worked examples. Pupils work 8 × 14 together at the board, then write three splits in their copy. The Class Challenge runs four products of rising difficulty, and the wrap tests whether the trick also holds over subtraction.

TEACHING MOVES

1. **Getting Started.** Pose 6 × 13 as a real head-maths attempt — give about ten seconds of quiet think-time, then take two or three hands and the route each pupil used. Do not name the rule yet; you want their splits as the raw evidence the next step builds on.
2. **Watch and Notice.** Walk the three rectangles one at a time, pointing at the two smaller pieces each time. After 4 × 25 (a clean 80 + 20 = 100), pause for a hands-up predict: 'which two rectangles will we get when we split 16 at the tens?' Only after the third example do you point to the on-screen rule and let the class finish it aloud.
3. **Try It Together.** Invite a pupil to the board to choose where to cut the 14 while the class agrees or corrects out loud. Watch for the pupil who multiplies the two parts together; revoice firmly: 'we add the two rectangles, we don't multiply them.' Keep it to this one product.
4. **Write the Splits in Your Copy.** Walk the room glancing that each split adds back to the whole product. The specific thing to catch: pupils who split into 10 + something but forget to work out the second partial product — they'll write 9 × 16 = 90 and stop.
5. **Class Challenge.** Pupils take turns at the board through the four products; the class confirms each answer before moving on. Keep it brisk — no re-teaching. On the last two (bigger tens parts), ask which rectangle is the largest each time and why.
6. **What Did We Notice?.** Don't confirm the subtraction case first — let pupils reason from the picture: start with the big 6 × 20 rectangle and cut off the 6 × 2 strip. Revoice that as 'the rule shares out over a minus the same way it shares out over a plus', then confirm 120 − 12 = 108.

COMMON MISCONCEPTIONS

⚠ A pupil splits 8 × 14 into 8 × 10 and 8 × 4 but then multiplies the two parts: 80 × 32 instead of 80 + 32.

Stop and point at the two rectangles on the area model. 'These are two pieces of one whole rectangle — we put them back together by adding.' Have the pupil read off each rectangle's area and add them aloud.

⚠ A pupil splits at the tens but drops the second partial product, writing 9 × 16 = 9 × 10 = 90.

Point at the smaller rectangle still sitting on screen. 'You've covered the 10-piece — what about this 6-piece over here?' Make them name 9 × 6 = 54 and add it before they call the answer done.

⚠ Pupils assume the subtraction version must be wrong because subtraction 'doesn't share out like adding does'.

Use the cut-off-the-strip picture: the big 6 × 20 rectangle minus the 6 × 2 strip leaves exactly 6 × 18. Let the picture do the convincing before you state the rule holds for minus too.

DIFFERENTIATION

EMERGING

- Keep the split at 10 + ones and stick to one-digit-times-teens (5 × 13, 6 × 12) while the rest move to twenties and thirties; pupils mirror the same two-rectangle picture in copy.
- Pre-draw the area-model rectangle with the tens cut already marked, so the pupil only fills in the two partial products rather than deciding the split.

DEVELOPING

- After the copybook splits, ask a pupil to find a second way to cut 6 × 24 (say 6 × 12 + 6 × 12) and check it lands on the same total.
- Give a missing-part variant: 7 × 13 = 7 × 10 + 7 × ?, and ask what the missing factor and partial product must be.

PROFICIENT

- While the Class Challenge runs, hand a fast finisher 7 × 99 and ask them to split it the easy way (7 × 100 − 7 × 1) and explain to the class why subtracting a strip is faster here than adding two pieces.
- Pose: can you split BOTH factors at once — 12 × 14 as four rectangles — and prove the four partial products still add to the whole?

○ **Cross-curricular:** Tie to Geography — pupils work out a hall's area in tiles by splitting a length like 14 m into 10 + 4 and adding the two rectangles.

ANSWER KEY

W1: commutative property

W2: commutative property

Q1: associative property

Q2: associative property

Q3: distributive property

Q4: distributive property

EXTENSION SHEET · STRETCH ANSWERS

S1: distributive property

S2: associative property

S3: commutative property