Lesson 104 ALGEBRA

Generalising number patterns and sequences

CURRICULUM ALIGNMENT

ALG.PRR.4a identify, explain and apply generalisations, including properties of operations, mathematical models and patterns.

ALG.PRR.4b represent mathematical structures in multiple ways, including verbal expressions, diagrams and symbolic representations.

INTERACTIVES Function Machine · challenge, display, explore

WHAT THIS LESSON TEACHES

A **sequence** follows a rule. Spotting how it grows lets you predict later terms and describe the pattern in general.

→ 3, 7, 11, 15... goes up 4 each time.

→ Term-to-term: 'add 4'. Position rule helps you jump ahead.

LESSON ARC

Open with 4, 7, 10, 13 on the board and the question 'what's the 100th term?' — let the class feel why listing is slow before any method. At the function-machine interactive, model the work-back on 1→4, 2→7, 3→10: the +3 step becomes ×3, then adjust +1 to fit. Pupils write three sequences with step and rule in their copies, then crack hidden-rule machines together at the board, closing on a non-constant-step sequence that breaks the trick.

TEACHING MOVES

1. **Getting Started.** Pose 'what's the 100th term?' and genuinely wait — take three or four hands-up ideas but reveal no method. If someone spots the +3, revoice it as 'so it grows by the same amount each time' and leave the leap-to-100 hanging deliberately.
2. **Watch and Notice.** Keep each machine's rule hidden — pupils deduce, you don't tell. Fully model the bridge on 4, 7, 10: find +3, link to ×position, then show 1×3=3 is one short of 4, so add 1 → ×3 then +1. Contrast with 2, 4, 6, 8 where the machine just doubles, to prove step and rule aren't always the same wording.
3. **Try It Together.** On 1→5, 2→7, 3→9, ask for the step first (+2), then 'what do we do to the input to land on the output?' — 1×2=2 but output is 5, so +3, rule is ×2 then +3. Have a pupil feed in a fresh number and the class predict the output before the machine reveals it — that prediction is the test.
4. **Write the Rules in Your Copy.** Walk the room checking pupils have circled the step and written the rule in words, not just a number. Prompt anyone stuck with 'how much does it grow each time?' — this is whole-class copybook practice, not marking.
5. **Class Challenge.** Keep board work brisk: warm up on the two single-step machines, then work the headline 6, 11, 16, 21 (step +5, 1×5=5 one short of 6, so ×5 then +1). For each, step then rule then a prediction on a fresh input before Check. Close by posing 3, 8, 15, 24 verbally only and asking 'why won't our usual trick work here?' — the step itself is changing (+5, +7, +9).
6. **What Did We Notice?.** Re-display 2, 4, 6, 8 and 5, 9, 13, 17 so pupils reason from what they see. Don't accept an asserted answer — make them test each claim on a real sequence. Revoice the resolution: 'the step tells us what to multiply by, but we still adjust to land on the first number, so both pupils are partly right.'

7. **What's Next.** Tie back to the opening: with rule position ×3 then +1, the 100th term of 4, 7, 10, 13 is 301 — no listing needed. That payoff is what made the rule worth hunting for.

COMMON MISCONCEPTIONS

⚠ Pupils say the step IS the rule — for 5, 9, 13, 17 they write 'the rule is +4' instead of ×4 then +1.

Send 1, 2, 3 through the machine on screen: ×4 alone gives 4, 8, 12, not 5, 9, 13. Show the gap of 1 at the first term, then add it back. The step tells you the multiplier; you still have to adjust to fit the first number.

⚠ Pupils assume every growing sequence must have a constant-step rule and try to force ×something then + something onto 3, 8, 15, 24.

Write the steps between terms on the board: +5, +7, +9. Ask 'is the step staying the same?' Once they see it's increasing, name that our usual trick only works when the step is constant — this one needs a different idea, coming later.

⚠ Pupils confuse position with value — when asked for the rule they multiply the term by 3 instead of the position by 3.

At the machine, line the inputs (1, 2, 3) directly above the outputs (4, 7, 10) and point: 'we feed in the position, not the number it gives back.' Have a pupil say aloud 'position one times three, then add one' before writing it.

DIFFERENTIATION

EMERGING

- Stay on single-step machines (just ×2, or just +4) in the copybook moment so these pupils nail step-spotting before any two-step adjustment.
- Give the step already circled on their copybook sequences so they only have to put the rule into words.

DEVELOPING

- After the copybook moment, ask them to use their rule to find the 10th term of 5, 9, 13, 17 without listing — then check by extending the sequence.
- Hand them a four-term sequence with a missing middle term and ask them to fill it using the rule, not the step.

PROFICIENT

- Once the headline machine is cracked, narrate a harder variant at the board: 'find me a sequence whose rule is ×3 then −2 — what are its first four terms?' working backwards from rule to sequence.
- Pull them ahead into building a function machine that gives outputs 1, 4, 9, 16 and ask why no constant-step rule fits — a bridge to the increasing-step idea from the closing sequence.

Cross-curricular: Tie to Geography — Irish bus-stop seating or GAA stand rows often grow by a constant number per row; pupils find the rule for total seats by row.

ANSWER KEY

W1: $\times 5$

W2: 23

Q1: 53

Q2: $+10$

Q3: 2

Q4: $\times 3 + 8$

EXTENSION SHEET · STRETCH ANSWERS

S1: 14

S2: $\times 3 + 6$

S3: -6

S4: $\times 6 + 4$

S5: $+11$

Investigation: Growing patterns — open-ended; scan pupils' working for valid solution paths rather than a single answer.